Letters

Comments on "The ZEPLS Program for Solving Characteristic Equations of Electromagnetic Structures"

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The ZEPLS routine [1] prepared by P. Lampariello and R. Sorrentino in the above paper[1] calculates the zeros of analytic function $f(z)$. Unfortunately, an additional difficulty is often introduced by the fact that the analytic function has polar singularities and that the poles are placed in the complex plane only by the zeros. In this case, the ZEPLS routine cannot be used successfully. On the other hand, the zeros and the poles create at that time the couples so that the regions with only one zero and one pole can be separated. The modification permits an adaptation of the ZEPLS routine for such a case.

We note that the formula (2) in [1] is a simplified form of a more general expression for Nth-order moments of the meromorphic function

$$s_N = \frac{1}{2\pi j}\oint_{+\partial D} z^N \frac{f'(z)}{f(z)}\,dz = \sum_{i=1}^{n_z} z_i^N - \sum_{k=1}^{n_p} p_k^N \tag{1}$$

where n_z, n_p are the numbers of zeros and poles, respectively, and z_i, p_k are the zeros and poles of $f(z)$, respectively.

The expression (1) for $N = 1,2$ and $n_z = n_p = 1$ gives

$$z_i = 0.5 \cdot \left(s_1 + \frac{s_2}{s_1}\right). \tag{2}$$

The formula (2) shows that for calculating the zero of $f(z)$, the knowledge of the moments s_1 and s_2 is sufficient. The suitable modification was introduced to the ZEPLS routine. Next, the modified program was used to calculate the propagation constant of the slot-line structure with a multilayered lossy substrate. An example of the numerical results is presented in [2, see fig. 4 and fig. 5].

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[1] P. Lampariello and R. Sorrentino, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 457–458, May 1975.

Reply[2] by P. Lampariello and R. Sorrentino[3]

As is well stated in the program description [1], the ZEPLS routine applies to regions where the function $f(z)$ is single-valued and holomorphic. In fact, the ZEPLS routine was originally developed for specific eigenvalue problems, such as the analyses of a waveguide inhomogeneously filled with a semiconductor [3] and of a lossy ferrimagnetic slab [4]. In such cases, as in many other similar problems, the characteristic equation can be represented by a holomorphic function $f(z)$ equated to zero.

Since the routine computes the moments s_N, it is quite obvious that it can be successfully modified in order to match the specific needs of different problems. Sometimes, for instance, $f(z)$ possesses polar singularities which are previously known, so that their contribution can be subtracted from the moments s_N. In other cases, when the polar singularities are unknown and are placed close to the zeros of $f(z)$, the modification suggested by W. Zieniutycz can be usefully applied. This permits the computation of one zero and one pole at the same time. In a similar manner, for instance, if two zeros and one pole are located inside the region (i.e., $n_z = 2$ and $n_p = 1$), using (1) with $N = 1,2,3$, it is easily seen that

$$z_1 + z_2 = \frac{2}{3}\,\frac{s_3 - s_1^3}{s_2 - s_1^2}.$$

The computation of z_1, z_2, and p_1 is then straightforward.

References

[1] P. Lampariello and R. Sorrentino "The ZEPLS program for solving characteristic equations of electromagnetic structure," *IEEE Trans. Microwave Theory Tech.*, vol MTT-23, pp. 457–458, May 1975.

[2] K. S. Grabowski, J. Mazur, and M. Kitliński "MIC waveguiding structures filled with anisotropic media," in *Proc. 10th Eur. Microwave Conf.*, (Warsaw, Poland), pp. 25–36.

[3] R. Sorrentino, "Exact analysis of rectangular waveguides inhomogeneously filled with a transversely magnetized semiconductor," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 621–625, Sept. 1976.

[4] F. Bardati and P. Lampariello, "The modal spectrum of a lossy ferrimagnetic slab," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 679–688, July 1979.

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